BANKING AND FINANCIAL FRAGILITY

A Baseline Model: Diamond and Dybvig (1983)

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Objective

- Want to develop a model to help us understand:
  - why banks and other financial institutions tend to have a maturity mismatch between their assets and liabilities
  - in what way(s) this maturity mismatch can create the type of financial crises we see in reality
- …and use this model to evaluate policy proposals
- Our model will be very simple in some dimensions
  - but we will get a remarkable amount of mileage out of it
- Readings:
  - Diamond & Dybvig (JPE, 1983)
  - Allen & Gale, chapter 3

Outline

1. The Environment
2. Autarky
3. The Efficient Allocation
4. Banking
5. Two Views of Financial Fragility
6. Summary

1. The Environment

1.1 Time and commodities

- 3 time periods
  - $t = 0, 1, 2$
- Single consumption good in each period

1.2 Economic agents

- Continuum of investors, $i \in [0,1]$
- Each is endowed with 1 unit of the good at $t = 0$
  - and nothing at $t = 1, 2$
- Each has utility function

$$\begin{Bmatrix} u\left(c_1^i\right) \\ u\left(c_2^i\right) \end{Bmatrix} \text{ if investor } i \text{ is } \begin{Bmatrix} \text{type 1 } - \text{"impatient"} \\ \text{type 2 } - \text{"patient"} \end{Bmatrix}$$

  - denote type by $\omega_i \in \Omega = \{1,2\}$
- At $t = 0$, investor does not know her type
  - learns type at $t = 1$
  - type is private information

Uncertainty

- Each investor will be impatient with probability $\lambda \in (0,1)$
- λ also = fraction of all investors who will be impatient
  - no aggregate uncertainty here
  - only uncertainty is about *which* investors will be impatient

Consumption plans

- A consumption plan for investor i is

$$c^i = \left(c_1^i, c_2^i\right) \in \mathbb{R}_+^2$$

1.3 Technologies

- Two assets for transforming $t = 0$ goods to later periods

- Storage:

$$1 \text{ unit at} \begin{Bmatrix} t = 0 \\ t = 1 \end{Bmatrix} \text{ yields } \begin{Bmatrix} 1 \text{ at } t = 1 \\ 1 \text{ at } t = 2 \end{Bmatrix}$$

- Investment:

$$1 \text{ unit at } t = 0 \;\text{ yields } \begin{Bmatrix} r < 1 \;\text{ at } t = 1 \\ R > 1 \;\text{ at } t = 2 \end{Bmatrix}$$

  - investment can only be started at $t = 0$
  - $(1 - r)$ = "liquidation cost"

2. Allocations under Autarky

- Suppose there is no trade
  - each investor divides her endowment at $t = 0$ between storage and investment
  - consumes the proceeds at either $t = 1$ or $t = 2$
- Let x = amount placed into investment
  - $(1 - x)$ is placed into storage
- Investor's objective: $\max_{\{x\}} \lambda u(c_1) + (1 - \lambda)u(c_2)$
- Feasibility constraints:

$$c_1 = rx + (1 - x) \quad = \quad 1 - (1 - r)x$$

$$c_2 = Rx + (1 - x) \quad = \quad 1 + (R - 1)x$$

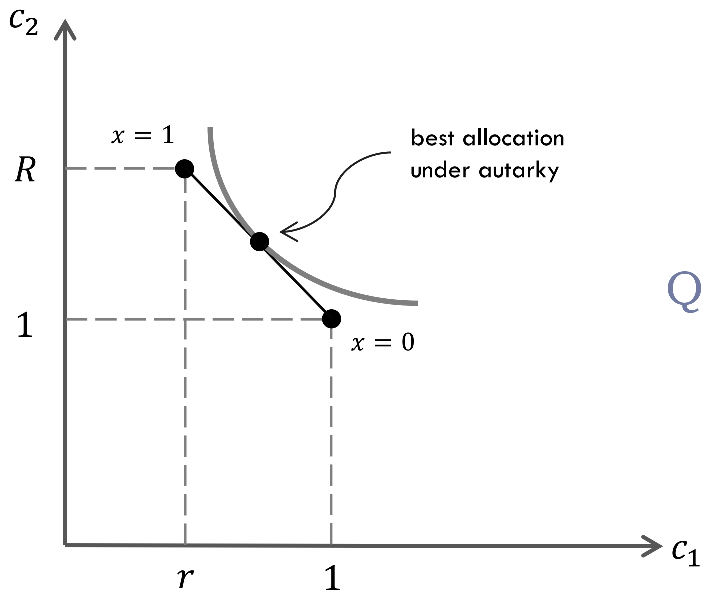
- Restating the investor's maximization problem:

$$\max_{\{x\in[0,1]\}} \lambda u(c_1)+(1-\lambda)u(c_2)$$

subject to

$$c_1 = 1-(1-r)x$$

$$c_2 = 1+(R-1)x$$



Q: Is this allocation Pareto optimal?

3. The (full information) efficient allocation

3.1 Definitions

- An allocation is a list of consumption plans:

$$\left\{\left(c_1^i, c_2^i\right)\right\}_{i\in[0,1]}$$

- An allocation is symmetric if

$$\left(c_1^i, c_2^i\right) = \left(c_1^j, c_2^j\right) \text{ for all } i, j$$

  - characterized by only two numbers

- Under full information, investors' preference types are observable (to the planner)

Q: What is the best symmetric allocation the planner can implement under full information?

3.2 Some properties of efficient allocations

- The efficient allocation of resources in this environment requires:
  - no investment should be liquidated at $t = 1$
  - no storage should be held until $t = 2$
    - recall that there is no aggregate uncertainty here

- In our notation:

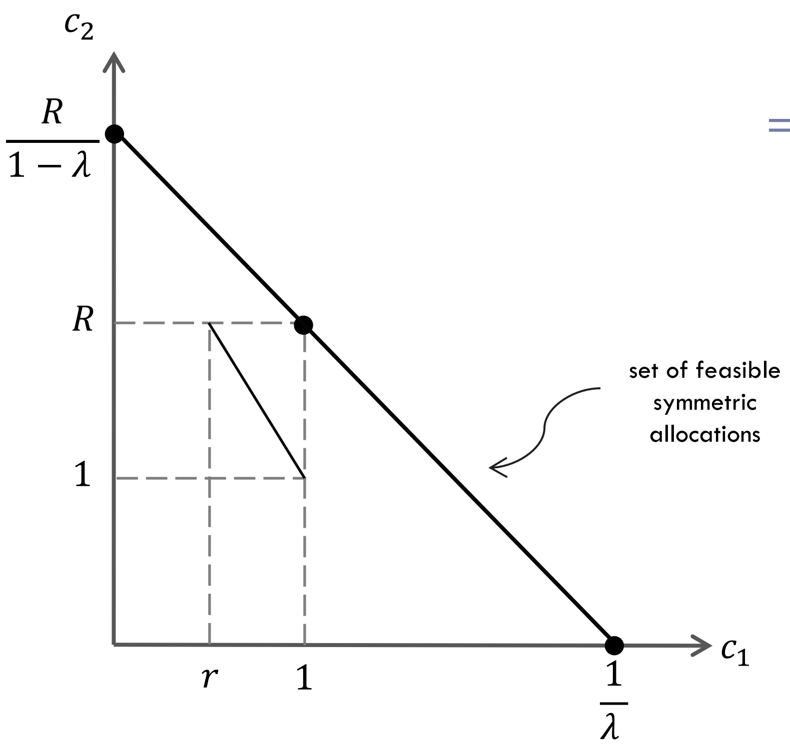
$$\begin{aligned} \lambda c_1 &= 1 - x \\ (1 - \lambda) c_2 &= Rx \end{aligned}$$

- Combining to eliminate x:

$$\lambda c_1 + (1 - \lambda) \frac{c_2}{R} = 1$$

- Repeating

$$\lambda c_1 + (1-\lambda)\frac{c_2}{R} = 1$$



⇒ The planner can do better than autarky (Why?)

3.3 Finding the best symmetric allocation

- The full-information efficient allocation solves

$$\max_{\{c_1, c_2\}} \lambda u(c_1) + (1-\lambda) u(c_2)$$

subject to $\lambda c_1 + (1-\lambda)\frac{c_2}{R} = 1$ multiplier = μ

- First-order conditions:

$$\lambda u'(c_1) = \lambda \mu$$

$$(1-\lambda) u'(c_2) = (1-\lambda)\frac{\mu}{R}$$

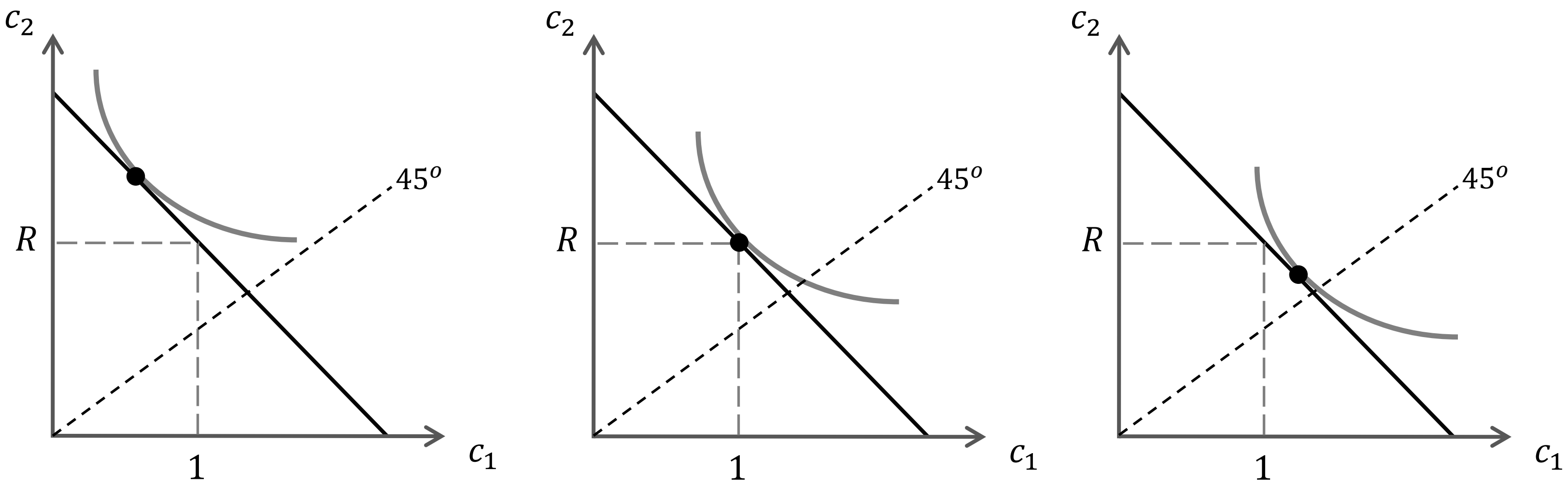
or

$$u'(c_1) = R u'(c_2)$$

- Solution:

$$(c_1^*, c_2^*) \text{ with } c_1^* < c_2^*$$

- Depending on the function u, we can have



- Efficient level of investment:

$$x^* = (1-\lambda)\frac{c_2^*}{R}$$

or $$(1-x^*) = \lambda c_1^*$$

Exercises

- We know (c_1^*, c_2^*) solves:

$$\max_{\{c_1,c_2\}} \lambda u(c_1) + (1-\lambda)u(c_2)$$

$$\text{subject to} \qquad \lambda c_1 + (1-\lambda)\frac{c_2}{R} = 1$$

- Find (c_1^*, c_2^*) for the following utility functions:
  - $u(c) = \ln(c)$ A: $(c_1^*, c_2^*) = (1, R)$
  - $u(c) = c$ (risk neutral) A: $(c_1^*, c_2^*) = (0, \frac{R}{1-\lambda})$

4. Banking

4.1 More on the environment

- Return to the case where types are private information
- Investors can meet at $t = 0$, but are isolated from each other at $t = 1$
  - cannot trade with each other
- Each investor can visit a central location at $t = 1$ before consuming
  - arrive one at a time
  - must consume when they arrive (ice cream on a hot day)
- These assumptions aim to capture transaction needs
  - when a consumption opportunity arises, investors cannot quickly sell illiquid assets

4.2 A banking arrangement

- Suppose a bank opens at $t = 0$, offers the following deal:
  - deposit at $t = 0 \Rightarrow$ you can withdraw at either $t = 1$ or $t = 2$ (your choice)
- Bank places a fraction x^* of its assets into investment
- Investors who choose $t = 1$ will receive c_1^*
  - as long as the bank has funds available
- Investors who choose $t = 2$ will receive an even share of the bank's matured assets
- These rules create a withdrawal game
  - each investor decides when to withdraw
  - payoffs depend on the choices made by all investors

4.3 Withdrawal strategies

- First: impatient investors will always withdraw at $t = 1$
  - do not value consumption at $t = 2$

⇒ We only need to determine what an investor will do *in the event she is patient*

- A withdrawal strategy is:

$$y_i \in \{1,2\}$$

  - where $y_i = t$ means withdraw in period t *when patient*

- More notation:
  - $y = \{y_i\}_{i \in [0,1]}$ is a complete profile of withdrawal strategies
  - y_{-i} = profile of strategies for all investors except i

4.4 Best responses

- Suppose an investor anticipates $y_{-i} = 2$
    - that is, all other investors will withdraw at $t = 2$ when patient
- What is her best response?
    - if she withdraws at $t = 1$: c_1^*
    - if she withdraws at $t = 2$: even share of matured investment
    - what is this even share worth?

matured investment

$$\frac{Rx^*}{1-\lambda} = \frac{(1-\lambda)c_2^*}{1-\lambda} = c_2^*$$

patient depositors

- We know $c_2^* > c_1^* \;\Rightarrow\;$ best response $y_i = 2$

4.5 Equilibrium

- A Nash equilibrium is a profile of withdrawal strategies y^* such that, for all i, y_i^* is a best response to y_{-i}^*.
  - focus on symmetric equilibria in pure strategies

Result 1: There is a Nash equilibrium with

$$y_i = 2 \quad \text{for all } i.$$

- In this equilibrium:
  - impatient investors withdraw at $t = 1$, receive c_1^*
  - patient investors withdraw at $t = 2$, receive c_2^*

  ⇒ implements the (full information) efficient allocation
    - even though types are private information (**!**)

4.6 Interpretations

- Notice what the bank is doing in this model
  - issuing demand deposits
  - while holding (some) illiquid assets
- Why is this activity socially desirable?
  - because investors face uncertainty about their liquidity needs
  - bank allows all investors to hold liquid claims
- This activity is often called “maturity transformation”
  - emphasize that this a productive activity
    - bank is “producing” liquidity
  - also called “fractional reserve banking”

- Suppose we construct the balance sheet of this bank

Assets		Liabilities	
Investment	Rx^*	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	E

  - note that investment is valued at “hold to maturity” price
- Equity (or “bank capital”) is defined as Assets – Liabilities

$$E \equiv Rx^* + (1 - x^*) - c_1^*$$

- A bank is said to be solvent if $E \geq 0$
  - by design, our banking arrangement is solvent
  - even though some of the bank’s assets are illiquid

5. Two views of financial fragility

- So far: it can be socially useful to have banks doing maturity transformation
    - allows all investors to hold liquid claims
    - while (partially) benefitting from the higher return on illiquid investment
- In practice, maturity transformation appears to be at the center of many financial crises
- What does our model say about the *fragility* of this banking arrangement?
- We can see two views of what happens during a crisis

5.1 Self-fulfilling bank runs

Q: Does the withdrawal game have other equilibria?

- Suppose investor i anticipates:

$$y_{-i} = 1$$

  - everyone else will "run" and withdraw at first opportunity

- What is her best response?
  - the bank will start liquidating investment …
  - should she join the run?

More generally:

- Find the best response of investor i to any profile y_{-i}

- For any y_{-i}, define:

$$e(y_{-i}) = \text{number of } t = 1 \text{ withdrawals that will be made by patient investors}$$

("extra" withdrawals at $t = 1$)

  - equals number of investors who have $y_i = 1$ *and* are patient
  - note: $e \in [0,1-\lambda]$

- To find best response of investor i:
  - compare expected payoffs of withdrawing at $t = 1$ and $t = 2$
  - both of these payoffs will depend on e

- If a patient investor chooses $t = 1$, she receives c_1^* …
  - … if (and only if) bank has funds available when she arrives
- If she chooses $t = 2$, she receives:
  - an even share of the bank's remaining (matured) assets
  - critical question: what is this even share worth?
- At $t = 2$, the bank will have:

$$\underbrace{1 - \underset{\text{storage}}{x^*} - \underset{\text{first } \lambda \text{ withdrawals}}{\lambda c_1^*}}_{= 0} + R\left(\underset{\text{investment}}{x^*} - \underset{\text{liquidated for extra } t = 1 \text{ withdrawals}}{e \frac{c_1^*}{r}} \right)$$

- Repeating: the bank will have

$$R\left(x^* - e\frac{c_1^*}{r}\right)$$

- Number of remaining investors: $1 - \lambda - e$

- An even share is worth:

$$c_2(e) = \max\left\{\frac{R\left(x^* - e\frac{c_1^*}{r}\right)}{1 - \lambda - e}, 0\right\}$$

Q: What does this function look like?

- Note:

$$c_2(0) = \frac{Rx^*}{1 - \lambda} = c_2^*$$

(as before)

- Assume

$$c_1^* > 1 - (1 - r)x^* \qquad \text{(A1)}$$

  - this condition implies the bank is “illiquid”
    - it cannot afford to give c_1^* to all investors at $t = 1$

- Then (you can verify):

$$\frac{dc_2(e)}{de} < 0$$

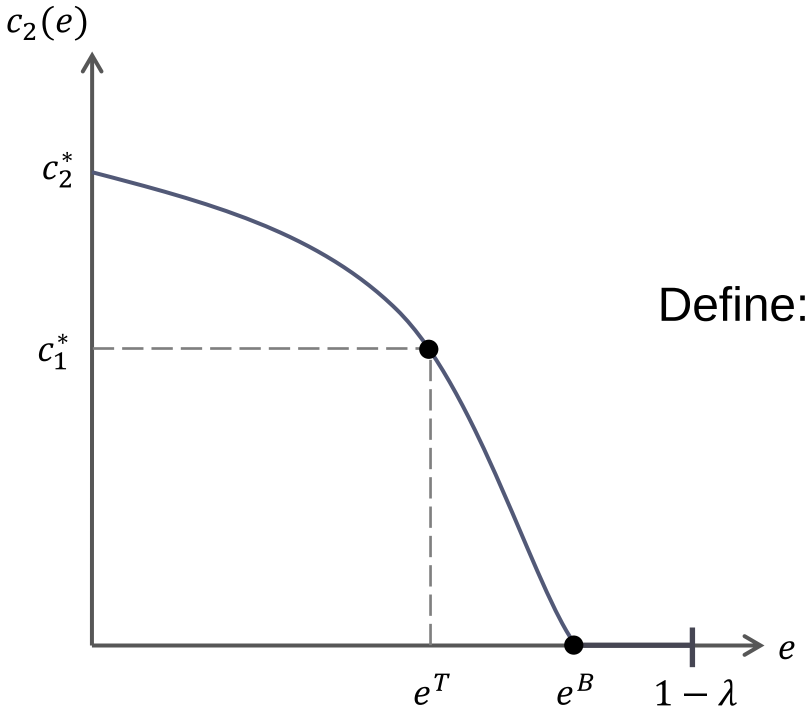
and

$$c_2(e) = 0 \quad \text{for some } \; e < 1 - \lambda$$

and

$$c_2(e) \text{ is strictly concave on } (0, e^B)$$

- Graphically:



Define: e^T (“threshold”) so that

$$c_2(e^T) = c_1^*$$

Define: e^B (“bankruptcy”) so that

$$c_2(e^B) = 0$$

- Summarizing investor *i*'s payoffs:

	$\boldsymbol{e < e^T}$	$\boldsymbol{e^T < e < e^B}$	$\boldsymbol{e > e^B}$
$t = 1$:	c_1^*	c_1^*	c_1^* or 0
$t = 2$:	$c_2(e) > c_1^*$	$c_2(e) < c_1^*$	0

- For any y_{-i}, the best response of investor i is:

$$\text{if } e(y_{-i}) \begin{Bmatrix} \leq \\ \geq \end{Bmatrix} e^T, \text{then } y_i = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$$

- If $y_{-i} = 1$, then $e(y_{-i}) = 1 - \lambda \; > e^T$, so …

$$\Rightarrow \text{best response is } y_i = 1$$

Result 2: There is also a Nash equilibrium with

$$y_i = 1 \text{ for all } i.$$

- This second equilibrium resembles the bank runs we have seen during financial crises
  - a “panic”, but with fully rational investors
  - nothing fundamental is wrong; bank is still solvent
  - the crisis is (simply) a result of self-fulfilling beliefs

- Another look at the balance sheet:

Assets		Liabilities	
Investment	rx^*	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	$\hat{E}$

- If assets are valued at liquidation prices, equity becomes

$$\hat{E} \equiv rx^* + (1 - x^*) - c_1^* < 0$$

hold to maturity prices

Assets		Liabilities	
Investment	Rx^*	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	E

liquidation prices

Assets		Liabilities	
Investment	rx^*	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	$\hat{E}$

- A bank is solvent if $E \geq 0$; otherwise it is insolvent (repeat)
- A bank is liquid if $\hat{E} \geq 0$; otherwise it is illiquid (new)

Results 1 and 2: When a bank is solvent but illiquid, the withdrawal game has (at least) two equilibria:

- $y_i = 2$ for all i: implements the planner's allocation (c_1^*, c_2^*)
- $y_i = 1$ for all i: a bank run

"self-fulfilling financial fragility"

Properties of the bank-run equilibrium:

- Fraction of investors served:

$$q \equiv \frac{\text{total assets}}{\text{amount per investor}} = \frac{1-(1-r)x^*}{c_1^*} < 1$$

- Expected utility in the bank-run equilibrium:

$$\begin{aligned} qu(c_1^*) + (1-q)u(0) &< u(qc_1^* + (1-q)0) \\ &= u(1-(1-r)x^*) \\ &< u(1) \\ &\leq u(\text{autarky}) \qquad (\,!\,) \end{aligned}$$

- Outcome is worse than having no bank at all

5.2 Bad news and bank runs

- Suppose at $t = 1$ investors learn the return on investment has fallen to $R_L < R$
  - unexpected shock (for simplicity)
  - banking contract (that is, x^*, c_1^*) is already fixed

- An investor who withdraws at $t = 2$ now receives

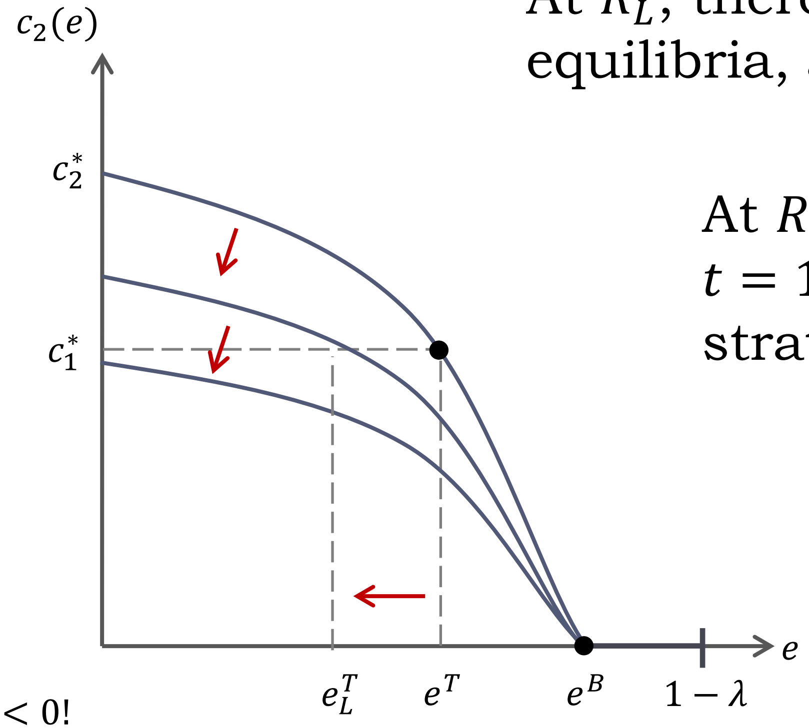
$$c_2(e) = \max \left\{ \frac{R_L \left(x^* - e \frac{c_1^*}{r} \right)}{1 - \lambda - e}, 0 \right\}$$

- Focus on:

$$c_2(0) = \frac{R_L x^*}{1 - \lambda}$$

- Consider two possibilities:
  - $R_{L'} < R_L < R$

At R_L, there are two equilibria, as before



$e_{L'}^T < 0$!

At $R_{L'}$, withdrawing at $t = 1$ is a dominant strategy !

⇒ A bank run is the <u>unique</u> Nash equilibrium

- How low must R_L be for withdrawing at $t = 1$ to become a dominant strategy?
- Start with $c_2(0) = \frac{R_L x^*}{1-\lambda}$
- Using $x^* = (1-\lambda)\frac{c_2^*}{R}$, we have

$$c_2(0) = \frac{R_L}{R} c_2^*$$

- Withdrawing at $t = 1$ is a dominant strategy if:

$$c_2(0) < c_1^*$$

or

$$R_L < \frac{c_1^*}{c_2^*} R \equiv \overline{R}_L$$

- Another view

Assets		Liabilities	
Investment	$R_L x^*$	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	E

  - "hold to maturity" value of investment has fallen
  - equity is now:

$$E = R_L x^* + (1 - x^*) - c_1^*$$

- (Verify:) $R_L < \bar{R}_L \Leftrightarrow E < 0$
  - if the loss is large enough to make the bank insolvent …
  - … withdrawing at $t = 1$ is a dominant strategy

Result 3: If $R_L < \overline{R}_L$, the *unique* Nash equilibrium strategy profile is

$$y_i = 1 \quad \text{for all } i.$$

- If the bank is insolvent, arrangement *necessarily* collapses
  - if c_1^* is close to c_2^*, the required losses would be very small

- Fraction of investors served in the run:

$$q = \frac{1-(1-r)x^*}{c_1^*} \quad \text{independent of } R_L!$$

  - Why? Because during a run, all investment is liquidated
  - same as when the run was based on self-fulfilling beliefs

- An example:
  - $u(c) = \ln(c)$ $\Rightarrow$ verify: $(c_1^*, c_2^*) = (1, R)$
  - also: $r = \frac{1}{2}$, $\lambda = \frac{1}{2}$ $\Rightarrow$ verify: $x^* = \frac{1}{2}$
  - then (verify) $\overline{R}_L = 1$
- Suppose $R_L = 0.99$
  - it is socially feasible to give all investors (almost) 1 unit
- The equilibrium allocation gives 1 to a fraction

$$q = \frac{1-(1-r)x^*}{c_1^*} = \frac{3}{4}$$

  - and nothing to the remaining 1/4 (much worse!)

6. Summary

Takeaways from Diamond & Dybvig (1983)

- Maturity transformation is socially useful …
  - D&D gave us a good model for thinking about where the value comes from
  - banks are in the business of "creating" liquidity
- … but makes banks fragile
- Two ways of thinking about this fragility
  - a bank that is solvent but illiquid is *susceptible* to a run
    - a loss of confidence – for whatever reason – leads to a run
  - a bank that is insolvent will *necessarily* have a run
    - small losses on a bank's assets can have large consequences

References and further reading

Franklin Allen and Douglas Gale (2007) *Understanding Financial Crises*, Oxford University Press.

- see especially Chapters 3 and 5

Diamond, Douglas W. and Phillip H. Dybvig (1983) "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91: 401-419.

Diamond, Douglas W. (2007) "Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model," Federal Reserve Bank of Richmond Economic Quarterly 93: 189-200.